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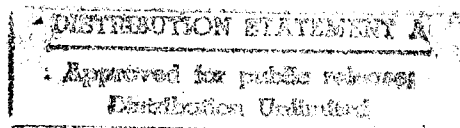
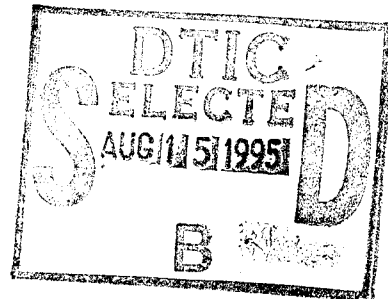
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AVERAGE NEUTRON WIDTH, RADIATIVE  
WIDTH, AND LEVEL SPACING FOR U-238 IN  
THE REGION 1 KV TO 400 KV

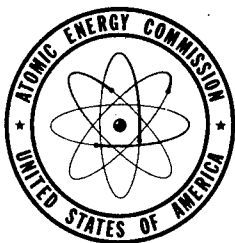
By  
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August 15, 1952

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General Electric Company  
KNOLLS ATOMIC POWER LABORATORY  
Schenectady, New York

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CONTENTS

	<u>Page</u>
Abstract . . . . .	3
Introduction . . . . .	4
Theory . . . . .	5
Determination of the Neutron Width. . . . .	5
Determination of the Average Radiative Width. . . . .	7
Determination of the Average Level Spacing D. . . . .	8
The Average $\Gamma_n$ , $\Gamma_\gamma$ , and D for U-238 . . . . .	8
Determination of the Average Neutron Width $\Gamma_n$ . . . . .	8
Determination of the Average Radiative Width. . . . .	9
Determination of the Average Level Spacing D. . . . .	11
Summary. . . . .	12
References . . . . .	13

ILLUSTRATION

KH-9A2294	Total Width for Neutron Emission vs. E . . . . .	10
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### ABSTRACT

The theory of nuclear reactions has been applied to U-238, and the nuclear parameters relevant to the proposed PPR (plutonium power reactor) have been deduced from the available experimental data. These nuclear parameters include the average neutron width, radiative width, and level spacing for U-238 in the range 1 kv to 400 kv, the approximate resonance parameters  $\sigma_0$ ,  $\Gamma$ , and  $\Gamma_n$  for the 6.6 ev resonance of U-238, and the incident neutron energy for which the neutron width equals the radiative width.

# AVERAGE NEUTRON WIDTH, RADIATIVE WIDTH, AND LEVEL SPACING FOR U-238 IN THE REGION 1 KV to 400 KV

J. A. Rich

## INTRODUCTION

The behavior of the proposed PPR (plutonium power reactor) will rest heavily on the nuclear properties of U-238. It is therefore of some consequence to investigate these nuclear properties. In particular, those nuclear parameters listed in the title are important for the calculation of the Doppler temperature coefficient of reactivity in the PPR.

As a guide to the detailed arguments required to establish the values of these parameters the following plan of work is presented:

1. The average neutron width  $\Gamma_n$  can be determined from

$$\Gamma_{nl} = 2k (\gamma_n)^2 T_l, \quad \Gamma_n = \sum_l (2l + 1) \Gamma_{nl} \quad (1)$$

where  $\Gamma_{nl}$  is the neutron width for emission of a neutron of energy  $E$  and angular momentum  $l$ ,  $\gamma_n^2$  is the reduced width, and  $k$  is the wave number of the neutron.  $T_l$  is a centrifugal barrier penetration factor analogous to the usual Coulomb barrier penetration factors.

The reduced width can be estimated from the resonance absorption integral for U-238.

2. The radiative width  $\Gamma_\gamma$  is determined from  $\sigma_0 \Gamma^2$ ,  $\sigma_0 \Gamma$  and  $\Gamma = \Gamma_\gamma + \Gamma_n$  for the lowest lying resonance.

3. The average level spacing  $D$  can be obtained from the ratio  $(\Gamma_\gamma/D)$  if  $\Gamma_\gamma$  is known.  $\Gamma_\gamma/D$  can be obtained from the expression for the average absorption cross section given by Wigner<sup>1</sup> as

$$\langle \sigma_a \rangle_{av} = \frac{4\pi^2}{k} \frac{(1 + kR)^2 \gamma_n^2 \Gamma_\gamma}{(\Gamma_\gamma + 2\gamma_n^2 k) D} \quad (2)$$

where  $D$  is the average spacing between levels of a determined spin and parity,  $R$  is the radius of the nucleus under consideration, and where  $\langle \sigma_a \rangle_{av}$ , the radiative capture cross section, has been measured. This relation is valid up to energies at which inelastic scattering takes place.

## THEORY

### Determination of the Neutron Width

The total neutron width for resonant elastic scattering  $\Gamma_n$  can be determined if the reduced width  $\gamma_{n1}^2$  is known (Eq. 1). However, it is possible to determine  $\gamma_{n1}^2$  from the value of the resonance absorption integral in the following way.

If we take the Breit-Wigner one-level formula as being applicable in the low energy resonance region we have for  $\ell = 0$

$$\sigma(n, r) = \frac{g\pi\kappa_1\Gamma_{n1}\Gamma_r}{(E - E_1)^2 + \Gamma^2/4}, \quad \kappa = \frac{\pi}{\sqrt{2mE}}, \quad g = \frac{1}{2} \left(1 \pm \frac{1}{2I + 1}\right) \quad (3)$$

where  $g$  is a statistical weight factor, and  $I$  is the spin of the target nucleus. It should also be noted that  $\Gamma$  is the full width at half maximum and that the subscript 1 refers to the 1'th resonance.

We have then as the contribution of the 1'th level to the resonance integral:

$$\left\{ \int_{E_c}^{\infty} \frac{\sigma dE}{E} \right\}_1 = g\pi\kappa_1\Gamma_r\Gamma_{n1} \int_{E_c}^{\infty} \frac{\kappa dE}{E \left[ (E - E_1)^2 + \frac{\Gamma_1^2}{4} \right]}$$

where  $E_c$  refers to the cadmium cut-off.

Setting  $x = 2 \frac{(E - E_1)}{\Gamma_1}$  we have

$$\left\{ \int \frac{\sigma dE}{E} \right\}_1 = \frac{g\pi\kappa_1\Gamma_r\Gamma_{n1}}{\sqrt{2m} E_1^{3/2}} \left( \frac{2}{\Gamma_1} \right) \int_{\frac{2(E_c - E_1)}{\Gamma_1}}^{\infty} \frac{dx}{1 + x^2}$$

For  $\Gamma_1 \ll E_1$  and for the lowest lying resonance well above the cadmium cut-off, we have

$$\int_{\frac{2(E_c - E_1)}{\Gamma_1}}^{\infty} \frac{dx}{1 + x^2} \approx \pi$$

For  $\ell = 0$ ,  $T_\ell = 1$ , and Equation 1 can be written  $\Gamma_{n1} = 2k_1\gamma_{n1}^2$ . Substituting and summing over 1, with the assumption that the level widths are small compared to the average level spacing so that each level contributes to the resonance integral as if it were alone, we have:

$$\int_{E_{Cd}} \frac{\sigma dE}{E} = \sum_i \left\{ \int_{E_{Cd}} \frac{\sigma dE}{E} \right\}_i = 4\pi^2 \gamma_n^2 g \sum_i \frac{\kappa_i \Gamma_i}{E_i \Gamma_i}$$

Further simplification results when we recall that in heavy nuclei and for low lying resonances,  $\Gamma \approx \Gamma_\gamma$ . Finally, we have to good approximation:

$$\int_{E_{Cd}} \frac{\sigma dE}{E} \approx 4\pi^2 g \gamma_n^2 \sum_i \frac{1}{\kappa_i E_i} = \frac{4\pi^2 g \gamma_n^2}{2.19 \times 10^9} \sum_i \frac{1}{E_i^{3/2}} \quad (4)$$

From the measured value of the resonance integral and a knowledge of the resonance energies it follows from Equation 4 that a value for  $\gamma_n^2$ , which is mainly determined by the lowest level, can be obtained. Using the relations  $\Gamma_{n\ell} = 2k\gamma_n^2 T_\ell$  and  $\Gamma_n = \sum_\ell (2\ell + 1) \Gamma_{n\ell}$  it is now possible to estimate the width for neutron emission as a function of energy.<sup>2</sup>

The "reduced width" around which our discussion has revolved refers to that factor in the transition probability which depends only on the interior of the nucleus. It is a function of the excitation energy  $E_{ex}$  of the compound nucleus but is independent of the neutron kinetic energy  $E_{kin}$  if  $E_{kin} \ll E_{ex}$ ; it is also independent of  $\ell$ . To obtain the actual transition probability or total neutron width, the reduced width must be multiplied by a factor which depends on the wave function in the external region. For S neutrons this factor is  $2k$  and for higher angular momenta it is  $2kT_\ell$ , where  $T_\ell$  is the centrifugal barrier penetration factor.

The  $T_\ell$ 's can be calculated exactly for neutrons and are listed below for the first few  $\ell$ 's.

$$T_0 = 1$$

$$T_1 = \frac{x^2}{1 + x^2}$$

$$x = kR$$

$$T_2 = \frac{x^4}{9 + 3x^2 + x^4}$$

$$T_3 = \frac{x^6}{225 + 45x^2 + 6x^4 + x^6}$$

$R$  is the radius of the target nucleus and  $k$  is the wave number of the incident neutron.  $R$  is best expressed by:\*

$$R = 1.50 \times 10^{-13} A^{1/3} \text{ cm}$$

---

\*NYO-636, p. 176



The factors  $T_l$  are calculated in the paper of Feshbach, Peaslee and Weisskopf.<sup>3</sup>

For a complete treatment of the concept of the "reduced width," reference should be made to E. Wigner<sup>1</sup> and to J. M. Blatt and V. Weisskopf.<sup>4</sup>

#### Determination of the Average Radiative Width

The radiative width  $\Gamma_\gamma$  for a single low lying resonance in a heavy nucleus where  $\Gamma = \Gamma_\gamma + \Gamma_n$ ,  $\Gamma_\gamma \gg \Gamma_n$  can be determined from the values for  $\sigma_0\Gamma$  and  $\sigma_0\Gamma^2$ .

$\sigma_0\Gamma$  can be determined from the reduced width in the following way: The Breit-Wigner one-level formula, Equation 3, reduces, at the resonance energy  $E_1$ , to

$$\sigma_{01}(n, \gamma) = \frac{4\pi g k_1^2 \Gamma_{n1} \Gamma_{\gamma 1}}{\Gamma_1^2}$$

where the subscript 1 refers to values at the resonance energy  $E_1$ . This can also be written as

$$(\sigma_0\Gamma)_1 = \frac{4\pi g}{k_1^2} \left(1 - \frac{\Gamma_{n1}}{\Gamma_1}\right) \Gamma_{n1} \quad (5)$$

where  $\Gamma_{n1} = 2k_1 \langle \gamma_n^2 \rangle_{av}$  and  $\langle \gamma_n^2 \rangle_{av}$  is obtained from Equation 4.

Insofar as  $\gamma_n^2$  and  $\Gamma_\gamma$  do not vary much from resonance to resonance these values may be considered as statistical average values.

The value of  $\sigma_0\Gamma^2$  for a particular resonance can be obtained from an analysis of the neutron time-of-flight data for the resonance in question.

An alternative procedure which may be valid in a few special cases is to determine the strength  $\sigma_0\Gamma^2$  of the lowest lying resonance from the value of the capture cross section at thermal energy by extrapolating the range of validity of the Breit-Wigner one-level formula down to thermal energy. How this follows can be seen by writing the Breit-Wigner expression in the following form:

$$\sigma(n, \gamma) = \frac{\sigma_0\Gamma^2 \left(\frac{E_0}{E}\right)^{1/2}}{\Gamma^2 + 4(E - E_0)^2} \quad (6)$$

where  $\Gamma^2 \ll 4(E - E_0)^2$  and  $\sigma(n, \gamma) \rightarrow \sigma_{th}(n, \gamma)$  for  $E \rightarrow E_{th}$ .

In general the capture cross section is made up of contributions from many levels, and these contributions are not simply additive but involve an interference between the resonances. This interference behaviour of absorption resonances

is dependent on the relative phases of the final states of the two or more resonances involved. Only in the case of heavy nuclei is there a possibility of the phase correlations between the many final states resulting from radiation of adjacent resonance levels, averaging to zero and consequently allowing a simple addition of contributions to the cross section from nearby resonances. More detailed arguments on the interference behavior of absorption resonances can be found in the literature.<sup>5</sup>

#### Determination of the Average Level Spacing D

The ratio  $(\Gamma_\gamma/D)$  can be obtained from Equation 2. It should be recalled that D is the average spacing between levels of a determined spin and parity.

The Expression 2 can also be written as:

$$\langle \sigma_a \rangle_{av} = \frac{2\pi^2}{k} (1 + kR)^2 \left( \frac{2\gamma_n^2 k}{\Gamma_\gamma + 2\gamma_n^2 k} \right) \frac{\Gamma_\gamma}{D}$$

In this form we see that  $\langle \sigma_a \rangle_{av}$  does not depend heavily on  $[2\gamma_n^2 k / (\Gamma_\gamma + 2\gamma_n^2 k)] = \xi$  for E large since  $\Gamma_\gamma \ll 2\gamma_n^2 k$  and  $\xi \rightarrow 1$ . As a result, the ratio  $\Gamma_\gamma/D$  should be as reliable as our knowledge of the radiative cross section at higher energies, since  $\sigma_a = \sigma(n, \gamma)$  in the region of interest.

To obtain D separately from the ratio  $(D/\Gamma_\gamma)$ , the value of  $\Gamma_\gamma$  determined from one of the low lying resonances is used.

The implication here is that  $\Gamma_\gamma$  remains constant over a wide energy range. This follows from theory and is supported by experimental evidence. The radiative width is not expected to change with energy since the binding energy is very much greater than the energy of the incident neutron, and because transition to a large number of states is always possible.

Experimental evidence here is most striking. In heavy nuclei  $\Gamma_{rad}$  is always of the order of 0.1 ev. In all cases in which  $\Gamma_{rad}$  could be measured directly it was found that:\*

$$0.02 \text{ ev} < \Gamma_{rad} < 0.15 \text{ ev} \quad (\text{heavy nuclei})$$

#### THE AVERAGE $\Gamma_n$ , $\Gamma_\gamma$ , AND D FOR U-238 (1 kv $\leftrightarrow$ 400 kv)

##### Determination of the Average Neutron Width $\Gamma_n$

The reduced neutron width can be obtained from Equation 4 in terms of the resonance energies (6.6 ev, 21 ev, 38 ev, ---) and the resonance absorption integral  $\int_{E_{Cd}} \sigma dE/E = 177$  barns for U-238. Taking into account that U-238 is an even-even nucleus we have  $g = 1$ . Substituting into Equation 4 we obtain:

$$\langle \gamma_n^2 \rangle_{av} = 1.33 \times 10^{-13} \text{ ev cm}$$

---

\*See Ref. 4, Table 2.1

Using the Relations 1 we can determine the width for neutron emission as a function of energy. This is shown in Figure KH-9A2294.

The value 177 barns for the "true" resonance integral, (the value of  $\int \sigma dE/E$  for no self-protection) is obtained by correcting the value 221 barns obtained by Hughes and Goldstein.<sup>6</sup> The value of the true resonance integral was obtained from

$$\int_U \frac{\sigma dE}{E} = \left( \frac{I_U}{I_{In}} \right)_{\text{latt}} \times \left( \frac{I_{In}}{I_U} \right)_{\text{T.C.}} \times \left( \frac{\sigma_U}{\sigma_{In}} \right)_{\text{th}} \int_{In} \frac{\sigma dE}{E}$$

where the I's refer to the initial counting rates of the foils irradiated either in the lattice or the thermal column of the graphite pile at Argonne. The  $\sigma$ 's refer to thermal cross sections. This can be written as:

$$\int \frac{\sigma dE}{E} = R \times K$$

where the term K is a constant for the experiment and R is a ratio of counting rates. The experimental value of R is given as  $5.40 \pm 0.04$ , the only source of error being the accuracy of counting. The value of K given by Hughes is 40.9 barns and is based on the following constants:

$$\int_{In} \frac{\sigma dE}{E} = 3000 \text{ barns}, \sigma_{In} = 190 \text{ barns}, \sigma_U = 2.59 \text{ barns}.$$

The value given above for the indium resonance integral is based on an integration of the  $\sigma$  vs. E curve given in the Metallurgy Lab project handbook.

The corrected value for K is 32.8 and is based on

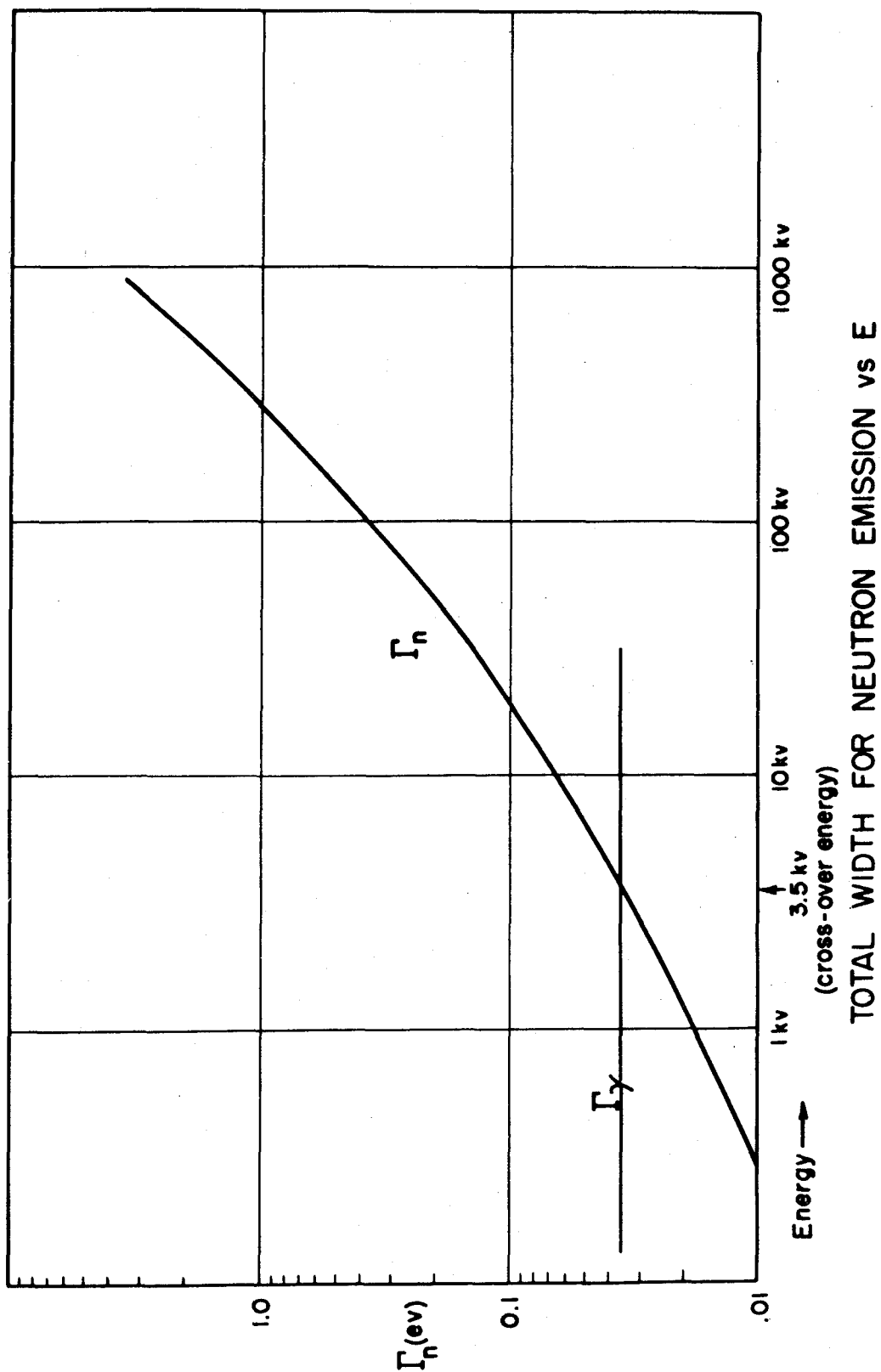
$$\sigma_{U-238} = 2.80 \text{ barns}, \sigma_{In-115} = 196 \text{ barns}, \int_{In-115} \frac{\sigma dE}{E} = 2294 \text{ barns}$$

The indium cross sections could have been given for total activation with the same final result for K. (Hughes actually measured the 54-min activity.) The value of  $(\int \sigma dE/E)_{In-115}$  is taken from the measurements of Harris, Muehlhause and Thomas.<sup>7</sup>

#### Determination of the Average Radiative Width

If  $\sigma_0 \Gamma$  and  $\sigma_0 \Gamma^2$  have been determined for the 6.6 ev resonance, then  $\Gamma$  and consequently  $\Gamma_\gamma$  are known for this resonance.

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Substituting  $(\Gamma_n)_{6.6} = 2k_{6.6} \langle \sigma_n^2 \rangle_{av} = 1.50 \times 10^{-3}$  ev into Equation 5 we obtain

$$(\sigma\Gamma)_{6.6 \text{ ev}} = 593 \left(1 - \frac{\Gamma_n}{\Gamma}\right)_{6.6} \text{ barns ev}$$

Since  $\Gamma_n \ll \Gamma$  in the resonance region we can take as a first approximation  $(\sigma_0\Gamma)_{6.6} \approx 593$  barns ev. Combining this with the value  $20.4$  barns  $(\text{ev})^2$  for  $(\sigma_0\Gamma^2)_{6.6 \text{ ev}}$ , a value which will be discussed further, we obtain  $(\Gamma)_{6.6} \approx 3.4 \times 10^{-2}$ . Substituting this back into the above expression we obtain as a second approximation  $(\sigma_0\Gamma)_{6.6} = 568$  barns (ev).

The value  $20.4$  barns  $(\text{ev})^2$  for the "strength" of the  $6.6$  ev resonance,  $(\sigma_0\Gamma^2)_{6.6}$ , has been obtained by Drs. R. Ehrlich and G. Roe of the KAPL theoretical group from an analysis of the Columbia neutron time-of-flight data.

We obtain then from  $\sigma_0\Gamma$  and  $\sigma_0\Gamma^2$  for the  $6.6$ -ev level the width  $\Gamma = 3.6 \times 10^{-2}$  ev and  $\sigma_0 = 1.5 \times 10^4$ .

Since  $\Gamma = \Gamma_n + \Gamma_\gamma$  in the resonance region we have finally the value  $\Gamma_\gamma = 3.5 \times 10^{-2}$  ev.

Returning to Figure KH-9A2294 we find that with  $\Gamma_\gamma$  determined, we can now locate the crossover point, the incident neutron energy for which  $\Gamma_n \sim \Gamma_\gamma$ . This is approximately  $3.5$  kv for U-238.

#### Determination of the Average Level Spacing D

In the following table are presented values of  $D/\Gamma_\gamma$  as a function of energy calculated from Equation 2. The values of  $\sigma(n,\gamma)$  for U-238 which correspond to the  $\langle \sigma_a \rangle_{av}$  in Wigner's Expression 2 are obtained from BNL-170.<sup>8</sup> This represents the best data available at this time. The experimental range covered is  $10$  kv to  $1.5$  Mev.

#### Average Level Spacing Calculated from the Radiative Capture Cross Section

<u>E (kv)</u>	<u>(D/<math>\Gamma_\gamma</math>)</u>	<u>D (ev)</u>
10	583	20.1
30	423	14.6
50	397	13.7
80	363	12.5
100	347	12.0
200	319	11.0
400	307	10.6

To obtain  $D$  separately from the ratio  $(D/\Gamma_\gamma)$  we use  $\Gamma_\gamma$  as determined from our analysis of the 6.6-volt resonance.  $D$  as a function of energy is also given in the table.

#### SUMMARY

The total width for neutron emission for U-238 has been determined as a function of energy up to 1 Mev.

The radiative width for U-238 is obtained from an analysis of the lowest lying resonance, 6.6 ev, and has the value 0.035 ev. The neutron width at this resonance is  $1.50 \times 10^{-3}$  ev. The total width is 0.036 ev and the corresponding capture resonance is 15,800 barns.

The crossover point (energy for which  $\Gamma \sim \Gamma_\gamma$ ) for U-238 occurs at about 3.5 kv.

The average level spacing in the region 10 kv  $\leftrightarrow$  400 kv is about 12 ev and is consistent with the measured level spacing in the resonance region.

No attempt was made to extend the energy range above 400 kv because of the complications introduced by the inelastic scattering of neutrons.

Finally, thanks are expressed to Dr. T. M. Snyder for many valuable discussions on the theory of nuclear reactions.

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